

Chapter 3 — Fraction Operations for Scaling and Sharing — Teacher Guide

Chapter at a glance

Lesson duration	4 lessons, 45–65 minutes each (see pacing table below)
Materials	Fraction strips (halves, thirds, fourths, sixths, eighths, tenths), paper strips, fraction circles, counters, measuring cups, small containers, a 0–2 number line, the five chapter diagrams
Launch question	“A recipe joins oats and seeds, a smaller batch scales flour, and juice is shared among people. What action is happening in each?”
Key representation	The common-unit strip, paired with labeled group diagrams for multiplication and division
Anticipated responses	Correct: names the action (join/separate/scale/share/group) before computing. Partial: computes correctly but can’t name the action. Incorrect: adds numerators and denominators straight across, regardless of the actual operation needed.
If the readiness check fails	Reteach equal-part identification and fraction-of-a-quantity from Chapter 2 before starting the Launch; do not skip the concrete strip-renaming step
Talk prompt	“What is happening to the quantity?”
Exit ticket	4 items: add with common unit, multiply as scaling, distinguish the two division meanings, state a magnitude/unit check (see Section 14)
Reteach / extend	Reteach: physical strips and small whole-number quantities only. Extend: solve using two representations, explain why the reciprocal appears in the division algorithm.

Purpose and pacing

Chapter 3 is the bridge between the meanings of fractions in Chapter 2 and the scaling, percent, ratio, and rate work that follows. The central instructional decision is to teach **meaning before algorithm**. Do not begin with “keep, change, flip” or a list of

procedures. Students need to understand what the operation does before the symbolic shortcut is introduced.

Lesson	Focus	Suggested time
Lesson 1	Readiness, launch, addition/subtraction, common units	45–60 minutes
Lesson 2	Multiplication as scaling and fraction of a quantity	45–60 minutes
Lesson 3	Division as sharing and grouping	45–60 minutes
Lesson 4	Worked-example critique, worksheets, Tripwire, and Exit Ticket	50–65 minutes

Materials

Prepare fraction strips marked in halves, thirds, fourths, sixths, eighths, and tenths; paper strips; fraction circles; counters; measuring cups; small containers; a 0–2 number line; and the chapter diagrams. Students should be able to physically join, separate, scale, share, and group quantities.

Teacher launch guidance

Begin with the three launch situations and do not name the operations immediately. Ask students to describe the action in everyday language. Record “join,” “scale,” “share,” and “group” on the board. Then connect each word to a mathematical operation.

If students use keywords mechanically, give a counterexample. For example, “A ribbon is left after metre is used” suggests subtraction, but “A student is left of the door” does not describe a fraction operation. Meaning must control the method.

Teacher prompts

Purpose	Prompt
Identify the action	“What is happening to the quantity?”
Establish a unit	“What size part does the denominator name?”
Connect to a model	“Show the operation before writing the equation.”
Support multiplication	“What is the fraction acting on?”
Support division	“Are we finding the amount in each group or the number of groups?”
Encourage estimation	“Should the result be less than, equal to, or greater than the starting quantity?”
Check units	“What does the answer measure?”
Diagnose an error	“What did the student assume about the units or the groups?”

Differentiation

Students needing concrete support

Use small whole-number quantities and physical materials. For addition and subtraction, use one strip per fraction and physically rename the units. For multiplication, make equal groups with counters. For division, alternate between sharing counters among people and counting how many fraction-sized groups fit.

Students needing language support

Use the frames:

- “The action is _____, so I will _____.”
- “The denominator names _____-sized parts.”
- “The numerator tells me _____.”
- “I know the answer is reasonable because _____.”
- “This is sharing division because _____.”
- “This is grouping division because _____.”

Students ready to extend

Ask students to solve a problem using two representations, compare multiplication by a fraction less than 1 with multiplication by a fraction greater than 1, create contexts for the same equation, and explain why the reciprocal appears in the symbolic division algorithm after the grouping meaning is established.

Formative assessment

During guided practice, record whether each student can identify the action, select a representation, preserve a common unit, calculate, and check. A correct answer with an incorrect model should be recorded as developing rather than secure because later proportional reasoning depends on the model being meaningful.

Answer-key policy

For open responses, accept equivalent models when units and quantities are preserved. Require students to distinguish $3 \div 4$ from $3 \div$, because the first asks for an amount per group while the second asks for a number of groups. When a student makes a procedural error, diagnose the underlying meaning before assigning more practice.

Revision Support: Expected Ideas and Teacher Moves

Use this compact guide with the new Cumulative Connection and Strategy Studio tasks. Accept equivalent representations when the relationship, units, and reasoning are correct.

Cumulative Connection: Explain the action in of 20. Find $+$ by naming the common unit. Describe the difference between division as sharing and division as grouping. Estimate whether $\times 18$ should be greater or less than 18.

Strategy Studio guidance: Expected ideas: $=$, so the sum is ; $\div =$ in both interpretations; multiplication by a fraction less than 1 shrinks, while a factor greater than 1 enlarges. Accept multiple stories if the units and action are consistent.

Transfer Bridge Teacher Guidance

Use the four tasks to distinguish imitation from transfer. Ask students to name the relationship, keep units visible, compare representations, and explain or repair an error. Accept equivalent methods when the structure and interpretation are correct.

Expected guidance: Familiar: . Altered: . Non-routine: mural paint is 6 litres, 9 litres remain, and each sign receives 3 litres, which is $\frac{1}{3}$ of the original. Repair: means sharing into 3 equal groups, so multiply by $\frac{3}{1}$; Student B is correct.

Record whether the difficulty is classification, representation, operation, computation, units, interpretation, or checking. Do not assign more routine practice until the source of the error is identified.

Discussion and evidence note

Ask students to name the relationship before calculating, compare at least two representations when appropriate, and revise an explanation after hearing another strategy. Record whether the difficulty is classification, representation, operation, computation, units, or checking.

Answer Key

Ready-to-Learn Check

1. .
2. , because $\frac{1}{4} = \frac{1}{4}$.
3. litre per person.
4. Less than 1.
5. 4.
6. It means division and, in a part-of-a-whole context, four equal parts make one whole.

Worksheet A

1. **Addition.** Likely wrong answer: "multiplication," because two fractions are named together. Misconception: assuming any two fractions in one problem must combine by multiplying. Next move: "are the oats and seeds being joined, or is one scaling the other?"
2. **Subtraction.** Likely wrong answer: "division," confusing "removed" with sharing. Misconception: keyword association without checking the action. Next move: "is a length being removed from the ribbon, or is the ribbon being split into groups?"
3. **Multiplication.** Likely wrong answer: "division," since "of the original" sounds like splitting into parts. Misconception: not recognizing "of" as a scaling cue. Next move: "is acting on the 20 grams, or is 20 being split into equal groups?"
4. **Division as sharing.** Likely wrong answer: "division as grouping," confusing the two division meanings. Misconception: not distinguishing "amount per group" from "number of groups." Next move: "does the question ask how much one person gets, or how many groups fit?"
5. **Division as grouping.** Likely wrong answer: "division as sharing," the same confusion in the opposite direction. Misconception: same as above. Next move: "does the question ask how much is in one bottle, or how many bottles are filled?"
6. $\frac{1}{4} = \frac{1}{4}$; $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$. Likely wrong answer: (adds numerators and denominators straight across). Misconception: treating numerator and denominator

as independent whole numbers to add. Next move: “what size piece is measured in? What size is ? Are they the same size?”

7. $= ; - = =$. Likely wrong answer: (subtracts numerators and denominators straight across). Misconception: the same straight-across error as #6, applied to subtraction. Next move: “rename both quantities in the same-sized unit before combining them.”
8. **12.** Likely wrong answer: 6 (finds one-third of 18 and stops, forgetting to multiply by 2). Misconception: the unit-fraction-only error — finding one group but not selecting the numerator’s worth of groups. Next move: “6 is one-third of 18 — what is two-thirds?”
9. **8 half-sized groups.** Likely wrong answer: 2 (divides 4 by 2 instead of recognizing that halves are smaller than wholes, so more of them fit). Misconception: assuming dividing by a fraction always shrinks the result, the same way dividing by a whole number does. Next move: “how many halves make one whole? How many halves make four wholes?”
10. **The denominators name different-sized units (thirds and fourths); they must be renamed to twelfths before adding.** Likely wrong answer: agrees that $+ =$, adding numerators and denominators separately. Misconception: the recurring straight-across addition error. Next move: “does $+$ name the same-sized parts? What common unit works for both?”
11. **Less than 20.** Likely wrong answer: “greater than 20,” assuming multiplication always increases a quantity. Misconception: not

recognizing that multiplying by a fraction less than 1 scales a quantity down. Next move: “is less than or greater than 1? What should that tell you?”

12. **Sharing asks how much is in one group; grouping asks how many groups fit.** Common flaw: describes only one of the two meanings, or uses “divide” for both without distinguishing the questions. Next move: ask for one example of each from the chapter.

Worksheet B

1. $= ; + =$. Likely wrong answer: (adds numerators and denominators straight across). Misconception: the recurring straight-across addition error. Next move: “what size piece is measured in? Rename it in tenths first.”
2. $.$ Likely wrong answer: (subtracts straight across without renaming to a common unit first). Misconception: not converting to eighths before subtracting. Next move: “rename as eighths, then subtract from $.$ ”
3. **20.** Likely wrong answer: 4 (finds one-sixth of 24 and stops). Misconception: the unit-fraction-only error. Next move: “4 is one-sixth of 24 — what is five-sixths?”
4. **6.** Likely wrong answer: 3 (finds one-third of 9 and stops, forgetting to double it). Misconception: unit-fraction-only error. Next move: “3 is one-third of 9 — what is two-thirds?”
5. **10.** Likely wrong answer: 2.5 (divides 5 by 2 instead of recognizing halves are smaller, so more fit). Misconception: assumes dividing always shrinks the result. Next move: “how many halves make 5 wholes?”

6. **4. Likely wrong answer:** a value from misapplying the fraction, such as dividing 3 by 3 and then by 4. *Misconception:* not recognizing how many three-quarter-sized groups fit in 3. *Next move:* “how many three-fourths make one whole? How many make three wholes?”
7. **= , so the total is cup.** *Likely wrong answer:* (straight-across addition). *Misconception:* the recurring straight-across error. *Next move:* “what size piece is measured in? Rename it in sixths.”
8. **6 pieces.** *Likely wrong answer:* (multiplies the two fractions instead of dividing to find how many pieces fit). *Misconception:* confusing “cut into pieces of this size” (division) with a fraction-of-a-fraction problem. *Next move:* “how many one-eighth pieces fit in three-fourths?”
9. **is less than 1, so scaling 20 by it must produce less than 20.** *Common flaw:* verifies with the specific numbers ($15 < 20$) instead of stating the general reasoning about the fraction’s size relative to 1. *Next move:* ask for the explanation without plugging in the numbers.
10. **The denominators are already the same (fifths); only the numerators are added: $+$ = .** *Likely wrong answer:* — exactly the error given in the problem, from adding the denominators together even though they already matched. *Misconception:* applying the “find a common denominator” habit even when the denominators are already equal, and adding them anyway. *Next move:* “are and already measured in the same-sized parts? What do you do with the denominator when it’s already shared?”
11. **The question asks how many halves fit in 4, so the answer is 8.** *Likely wrong answer:* agrees with the student’s answer of 2, from dividing 4 by 2 instead of by . *Misconception:* confusing “divide by 2” with “divide by one-half” — treating the fraction as if it were the whole number 2. *Next move:* “how many half-sized groups fit in one whole? In four wholes?”
12. **Answers vary; must describe four wholes divided into half-sized groups (for example, “how many half-litre bottles can be filled from 4 litres?”).** *Common flaw:* creates a situation that actually describes (splitting 4 into 2 groups) rather than (counting half-sized groups). *Next move:* check the situation asks “how many half-sized groups fit,” not “what do 2 groups each get.”

Worksheet C

1. **= and = ; total = 1 km.** *Likely wrong answer:* (straight-across addition of numerators and denominators). *Misconception:* the recurring straight-across error. *Next move:* “rename both distances using twelfths before adding.”
2. **= ; - = = remains.** *Likely wrong answer:* (straight-across subtraction, or adds instead of subtracts). *Misconception:* not renaming to a common unit first. *Next move:* “what size parts are five-sixths and one-third measured in? Find a shared size.”
3. **$\times 40 = 24$ g.** *Likely wrong answer:* 8 g (finds one-fifth of 40 and stops, forgetting to multiply by 3). *Misconception:* the unit-fraction-only error. *Next move:* “8 g is one-fifth of 40 — what is three-fifths?”

4. **$6 \div = 8$ bottles.** Likely wrong answer: 4.5 (multiplies 6 by instead of dividing). Misconception: confusing “how many fit” (division) with “what fraction of the amount” (multiplication). Next move: “does the question ask for a fraction of six litres, or how many three-fourths-litre bottles six litres makes?”
5. **$3 \div 5 =$ litre per person.** Likely wrong answer: (reverses which number is being shared and which is the number of groups). Misconception: dividend/divisor order confused — the same error as writing as . Next move: “what total amount is being shared, and among how many people?”
6. **$\div$ asks how many sixths fit in two-thirds; $=$, so the answer is 4.** Likely wrong answer: — exactly the error given in the problem, from multiplying denominators together instead of finding a common unit and counting groups. Misconception: treating fraction division like fraction multiplication. Next move: “how many sixths make two-thirds? That tells you how many groups of one-sixth fit.”
7. **$\times =$ litre.** Likely wrong answer: adds instead of multiplies, giving . Misconception: not recognizing “three-fourths of a poster’s worth” as a scaling action. Next move: “is three-fourths of a poster more paint or less paint than a whole poster’s worth?”
8. **Answers vary; must give one genuine context for each of addition, subtraction, multiplication, and division.** Common flaw: gives two contexts that are really the same action worded differently, most often confusing sharing and grouping division. Next move: check that join, separate, scale, and share/group are all genuinely represented.
9. **Using twelfths: $=$, $=$, total $=$; a number line or benchmark check is also acceptable.** Common flaw: the two methods produce different answers, revealing an error in one of them, but the student doesn’t notice or reconcile the mismatch. Next move: “do both methods agree? If not, which step is different?”
10. **Dividing by a fraction less than 1 counts smaller groups, so more of them fit — the answer is larger than the starting number.** Common flaw: restates the rule without explaining why, or gives an example without the general reasoning. Next move: ask for the explanation in terms of group size, not just a memorized rule.
11. **$\times 5 =$ gallons. The total number of rooms in the house (12) and the number of days (3) were not needed to find the paint used.** Likely wrong answer: works the 12 or the 3 into the calculation somehow, or divides by 12 to find “paint per total house.” Misconception: assuming every number in a word problem must be used. Next move: “what does the question actually ask for? Which numbers answer that?”
12. **Answers vary; must describe a situation where is a starting amount and is removed from it — a separating action, not joining, scaling, sharing, or grouping.** Common flaw: writes a context but describes joining or scaling instead of separating (for example, “combining two amounts” rather than “removing part of an amount”). Next move: ask the student to name the action explicitly

before checking the context matches it.

Extend Your Thinking

1. $6 \div 2 = 3$, while $6 \div = 12$; the divisor describes the group size.
2. Yes, $\times = 1$, so the original quantity is restored.
3. Yes; for example, $\div = .$
4. Answers vary, but one must describe sharing and one scaling.
5. A denominator identifies the size of the unit being counted; it is not an arbitrary number.

Tripwire and Exit Ticket

Tripwire: $+$ $=$ after renaming both in sixths.

Exit Ticket: $=$;

answer ;

of $16 = 12$;

$4 \div$ counts eight halves, while $4 \div 2$ makes two groups of 2; checks may use magnitude, units, inverse operation, or representation.

